

### Exercise 4.1

Q.1 Prove that:

- (a)  $\sin(A + B) + \sin(A - B) = 2 \sin A \cdot \cos B$
- (b)  $\cos(A + B) + \cos(A - B) = 2 \cos A \cdot \cos B$

Q.2 Find the value of:

- (a)  $\sin(30^\circ + 45^\circ)$
- (b)  $\sin(45^\circ - 30^\circ)$
- (c)  $\cos(30^\circ + 45^\circ)$
- (d)  $\cos(45^\circ - 30^\circ)$
- (e)  $\sin(60^\circ - 45^\circ)$
- (f)  $\sin(60^\circ + 45^\circ)$
- (g)  $\cos(60^\circ + 45^\circ)$
- (h)  $\tan(45^\circ - 30^\circ)$

Q.3 Find the value of:

- (a)  $\sin 15^\circ$
- (b)  $\tan 15^\circ$
- (c)  $\sin 75^\circ$
- (d)  $\sin 75^\circ$
- (e)  $\cos 75^\circ$
- (f)  $\tan 75^\circ$
- (g)  $\sin 105^\circ$
- (h)  $\cos 105^\circ$
- (i)  $\tan 105^\circ$
- (j)  $4 \cos 75^\circ \cdot \sin 105^\circ$
- (k)  $\sin 18^\circ$

Q.4 Prove that:

- (a)  $\tan 15^\circ + \cot 15^\circ = 4$
- (b)  $\sin 75^\circ - \sin 15^\circ = 1/\sqrt{2}$
- (c)  $\sin 75^\circ + \cos 15^\circ = (1 + \sqrt{3}) / 2\sqrt{3}$
- (d)  $\sin 105^\circ + \cos 15^\circ = (\sqrt{3} + 1) / \sqrt{2}$
- (e)  $\cos 105^\circ + \cos 15^\circ = 1/\sqrt{2}$
- (f)  $\cot 75^\circ - \tan 105^\circ = 4$
- (g)  $\sin 75^\circ - \cos 75^\circ = 1/\sqrt{2}$
- (h)  $\sin 15^\circ - \cos 15^\circ = -1/\sqrt{2}$
- (i)  $\sin 105^\circ + \cos 75^\circ = \sqrt{3/2}$
- (j)  $\tan 75^\circ + \cot 75^\circ = 4$
- (k)  $\sin 15^\circ - \sin 75^\circ = -1/\sqrt{2}$
- (l)  $\cos 18^\circ - \sin 18^\circ = \sqrt{2} \cdot \sin 27^\circ$
- (m)  $\sin 100^\circ - \sin 20^\circ = \sin 40^\circ$

Q.5 Prove that:

- (a)  $\sin(A + 45^\circ) + \frac{1}{\sqrt{2}}(\sin A + \cos B)$
- (b)  $\cos(45^\circ + A) + \frac{1}{\sqrt{2}}(\cos A - \sin A)$
- (c)  $\sin(45^\circ + A) = \frac{1}{\sqrt{2}}(\cos A - \sin B)$
- (d)  $\cos(45^\circ + A) - \sin(45^\circ + A) = \sqrt{2} \cos A$
- (e)  $\sin(45^\circ + \theta) - \sin(45^\circ - \theta) = \sqrt{2} \sin \theta$
- (f)  $\cos(45^\circ + \theta) - \sin(45^\circ - \theta) = -\sqrt{2} \sin \theta$
- (g)  $2 \sin(45^\circ + x) \cdot \sin(45^\circ - x) = \cos 2x$
- (h)  $1 - \sin 2(45^\circ - \alpha) = \sin 2\alpha$
- (i)  $1 - \sin 2A = 2 \sin 2(45^\circ - A)$

Q.6 If  $\sin A = 3/5$  and  $\sin B = 12/13$ , find the value of the following.

- (a)  $\sin(A + B)$
- (b)  $\sin(A - B)$
- (c)  $\cos(A + B)$
- (d)  $\cos(A - B)$
- (e)  $\tan(A + B)$
- (f)  $\tan(A - B)$
- (g)  $\sin(A + B) + \sin(A - B)$
- (h)  $\sin(A + B) - \sin(A - B)$
- (i)  $\sin(A + B) + \cos(A - B)$
- (j)  $\cos(A + B) - \cos(A - B)$
- (k)  $\sin(A + B) + \cos(A + B)$
- (l)  $\sin(A + B) - \cos(A + B)$

Q.7 If  $\sin \alpha = 3/5$  and  $\cos \beta = 12/13$ , find the value of the following.

- (a)  $\sin(A + B)$       (b)  $\cos(A + B)$       (c)  $\tan(A + B)$

Q.8 If  $\tan \theta = 5/6$  and  $\tan \beta = 1/11$ , prove that:  $\theta + \beta = \pi/4$

Q.9 If  $\sin A = 3/5$  and  $\cos B = 5/13$ , prove that:

- (a)  $\sin(A + B) = 63/65$       (b)  $\cos(A - B) = 56/65$

Q.10 If  $\tan A = p$  and  $\tan B = 1/p$ , prove that:  $A + B = \pi/2$

Q.11 If  $\sin A = 1/\sqrt{10}$  and  $\sin B = 1/\sqrt{5}$ , prove that:  $A + B = \pi/4$

Q.12 If  $\tan(A - B) = 40$  and  $\cot B = 5/6$ , find the value of  $\tan A$ . (Escosol 2063)

Q.13 Prove that:  $2 \sin 2(\pi/8 + A/2) - 2 \sin 2(\pi/8 - A/2) = \sqrt{2} \sin A$ .

Prove that the following identities:

$$\text{Q.14 } \cot(A - B) = \frac{\cot B \cdot \cot A + 1}{\cot B - \cot A}$$

$$\text{Q.15 } \cot(A + B) = \frac{\cot B \cdot \cot A - 1}{\cot B + \cot A}$$

$$\text{Q.16 } \sin(A + B) \cdot \sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$

$$\text{Q.17 } \cos(A + B) \cdot \cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$$

$$\text{Q.18 } \tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\text{Q.19 } \tan(A - B) = \frac{\sin(A - B)}{\cos(A - B)} = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

#### Exercise 4.2

Q.1 If  $\sin\theta = 4/5$ , find the value of  $\cos 2\theta$  and  $\tan 2\theta$ .

Q.2 If  $\sin A = 3/5$ , find the value of  $\sin 2A$ ,  $\cos 2A$  and  $\tan 2A$ .

Q.3 If  $\sin A = 5/13$ , find the value of  $\sin 2A$ ,  $\cos 2A$  and  $\tan 2A$ .

Q.4 If  $\cos A = 12/13$ , find the value of  $\sin 2A$ ,  $\cos 2A$  and  $\tan 2A$ .

Q.5 If  $\tan A = 3/4$ , find the value of  $\sin 2A$ ,  $\cos 2A$  and  $\tan 2A$ .

Q.6 If  $\sin\theta = 1/2$ , find the value of  $\sin 3\theta$ ,  $\cos 3\theta$  and  $\tan 3\theta$ .

Q.7 If  $\tan\theta = 3/4$ , find the value of  $\sin 3\theta$ ,  $\cos 3\theta$  and  $\tan 3\theta$ .

Q.8 If  $\sin\alpha = 3/4$ , find the value of  $\cos 2\alpha$  and  $\sin 3\alpha$ .

Q.9 If  $\cos\theta = 6/5\sqrt{2}$ , prove that:  $\cos 2\theta = 11/25$

Q.10 If  $\sin A = 1/4$ , prove that:  $\cos 2A = 7/8$  and  $\sin 2A = \sqrt{15}/8$

Q.11 Express  $\sin 3A \cdot \cos^2 A$  in terms of  $\sin A$ .

Q.12 Express  $\cos 3A$  in terms of  $\cos A$ .

Q.13 Express  $\sin 3A$  in terms of  $\sin A$ .

Q.14 Express  $\tan 3A$  in terms of  $\tan A$ .

Q.15 Express  $\sin 2A$  in terms of  $\sin A$ .

Q.16 Express  $\sin 2A$  in terms of  $\cos A$ .

Q.17 Express  $\cos 2A$  in terms of  $\cos A$ .

Q.18 Express  $\cos 2A$  in terms of  $\cos A$ .

Q.19 Express  $\tan 2A$  in terms of  $\tan A$ .

Q.20 Express  $\cos 2A$  in terms of  $\tan A$ .

Q.21 Express  $\sin 2A$  in terms of  $\tan A$ .

Q.22 Express  $\cot 2A$  in terms of  $\cot A$ .

Q.23 Express  $\cot 3A$  in terms of  $\cot A$ .

Q.24 If  $\cos 2\theta = -7/25$ , find the value of  $\sin \theta$ .

Prove the following relations

$$\text{Q.25 } \cos A = \sqrt{\frac{1 + \cos 2A}{2}}$$

$$\text{Q.26 } \sin A = \sqrt{\frac{1 - \cos 2A}{2}}$$

$$\text{Q.27 } \tan A = \sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}}$$

$$\text{Q.28 } \cot A = \sqrt{\frac{1 + \cos 2A}{1 - \cos 2A}}$$

$$\text{Q.29 } \frac{1 - \cos 2A}{\sin 2A} = \tan A$$

$$\text{Q.30 } \frac{\sin 2A}{1 + \cos 2A} = \tan A$$

$$\text{Q.31 } \frac{\cos 2A}{1 + \sin 2A} = \frac{\cos A - \sin A}{\cos A + \sin A} = \frac{1 - \tan A}{1 + \tan A}$$

$$\text{Q.32 } 1 - \cos 2A = \tan^2 A$$

$$1 + \cos 2A$$

$$\text{Q.33 } \frac{2 - \sec^2 A}{\sec^2 A} = \frac{1}{\sec 2A}$$

$$\text{Q.34 } \frac{1 - \sin 2A}{1 + \sin 2A} = \frac{1 - \tan A}{1 + \tan A}$$

$$\text{Q.35 } \frac{1 - \cos A}{2 \cos A} = \sin A \cdot \tan A$$

$$\text{Q.36 } \frac{1 + \sin 2A}{\cos 2A} = \frac{\cot A + 1}{\cot A - 1} = \frac{\cos A + \sin A}{\cos A - \sin A}$$

$$\text{Q.37 } 2 \operatorname{cosec} 2\theta = \tan \theta + \cot \theta$$

$$\text{Q.38 } 2 \cot \alpha = \cot \alpha - \tan \alpha$$

$$\text{Q.39 } \frac{\cos \theta}{\cos \theta - \sin \theta} - \frac{\cos \theta}{\cos \theta + \sin \theta} = \tan 2\theta$$

$$\text{Q.40 } \frac{1 - \cos 2A}{1 + \cos 2A} + \frac{\sin 2A}{\sin 2A} = \tan A$$

$$\text{Q.41 } \frac{\cot A - \tan A}{\cot A + \tan A} = \cos 2A$$

$$\text{Q.42 } \frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} = \tan \theta$$

$$\text{Q.43 } \tan 2\alpha + \sin 2\alpha = \frac{4 \tan \alpha}{1 - \tan 4\alpha}$$

$$\text{Q.44 } \tan 2\alpha + \sec 2\alpha = \frac{\cos \alpha + \sin \alpha}{\cos \alpha - \sin \alpha}$$

$$\text{Q.45 } (\sin \theta + \cos \theta)^2 = 1 + \sin 2\theta$$

$$\text{Q.46 } 2 \sin(\pi^c/4 - \theta) \sin(3\pi^c/4 + \theta) = 1 - \sin 2\theta$$

$$\text{Q.47 } \cos 2\alpha - \cos 6\alpha = 4 \sin^2 2\alpha \cdot \cos 2\alpha$$

Long Questions

Prove the following identities.

$$Q.1 \ 2\sin^2(45^\circ - A) = 1 - \sin 2A$$

$$Q.2 \ \cos^2(45^\circ - A) - \sin^2(45^\circ - A) = \sin^2 A$$

$$Q.3 \ \cot(45^\circ + A) - \tan(A - 45^\circ) = \sin^2 A$$

$$Q.4 \ \frac{1 + \tan^2(45^\circ - \theta)}{1 - \tan^2(45^\circ - \theta)} = \operatorname{cosec} \theta$$

$$Q.5 \ \tan(45^\circ + \theta) = \frac{\cos 2\theta}{1 - \sin 2\theta}$$

$$Q.6 \ \tan(45^\circ + \alpha) + \tan(\alpha - 45^\circ) = 2\tan \alpha$$

$$Q.7 \ \tan(\alpha + 45^\circ) - \tan(\alpha - 45^\circ) = 2\sec \alpha$$

$$Q.8 \ \frac{1 - \tan^2(\pi/4 - \theta)}{1 + \tan^2(\pi/4 - \theta)} = \sin 2\theta$$

$$Q.9 \ \tan(\pi/4 + A) + \tan(\pi/4 - A) = 2\sec 2A$$

$$Q.10 \ \tan(\pi/4 - \alpha) = \frac{1 - \sin 2\alpha}{\cos 2\alpha} = \frac{1 - \tan \alpha}{1 + \tan \alpha}$$

$$Q.11 \ \frac{1 - \sin 2A}{1 + \sin 2A} = \frac{(1 - \tan A)^2}{(1 + \tan A)^2} = \frac{(\cot A - 1)^2}{(\cot A + 1)^2}$$

$$Q.12 \ \sin 2A + \tan 2A = \frac{4\tan A}{1 - \tan 4A}$$

$$Q.13 \ \frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2$$

$$Q.14 \ \frac{\sin 5\theta}{\sin \theta} - \frac{\cos 5\theta}{\cos \theta} = 4\cos 2\theta$$

$$Q.15 \ \operatorname{Cosec} 10^\circ - \sqrt{3} \sec 10^\circ = 4$$

$$Q.16 \ \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$$

$$Q.17 \ \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = 4$$

$$Q.18 \ 4(\cos^3 20^\circ + \sin^3 10^\circ) = 3(\cos 20^\circ + \sin 10^\circ)$$

$$Q.19 \ 4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\cos 10^\circ + \sin 20^\circ)$$

$$Q.20 \ \cos^2 \theta + \sin^2 \theta \cdot \cos 2\beta = \cos^2 \beta + \sin^2 \beta \cdot \cos 2\theta$$

$$Q.21 \ \tan(A + B) = \frac{\sin^2 A - \sin^2 B}{\sin A \cdot \cos A - \sin B \cdot \cos B}$$

$$Q.22 \ \frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} = 1 - \frac{1}{2} \sin 2A$$

$$Q.23 \ (2\cos \theta + 1)(2\cos \theta - 1)(2\cos 2\theta - 1) = 2\cos 4\theta + 1$$

$$Q.24 \ \tan(60^\circ + A) \tan(60^\circ - A) = \frac{2\cos 2A + 1}{2\cos 2A - 1}$$

$$Q.25 \ \operatorname{Cosec}^2 A - \sec^2 A = 4\operatorname{cosec} 2A \cdot \cot A$$

$$Q.26 \ \sin \theta \cdot \sin(60^\circ - \theta) \cdot \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$$

$$Q.27 \ \cos \theta \cdot \cos(60^\circ - \theta) \cdot \cos(60^\circ + \theta) = \frac{1}{4} \cos 3\theta$$

$$Q.28 \ \cot A + \cot(60^\circ + A) - \cot(60^\circ - A) = 3\cot 3A$$

$$Q.29 \ \tan A \cdot \tan(45^\circ + A) \tan(45^\circ - A) = \tan 3A$$

$$Q.30 \ \sec^2 \theta (1 + \sec 2\theta) = \sec 2\theta$$

$$Q.31 \ (1 + \tan \theta - \sec \theta) + (1 + \tan \theta + \sec \theta) = \sin 2\theta \cdot \sec^2 \theta$$

$$Q.32 \ \frac{\cos^3 \theta - \cos 3\theta}{\cos \theta} = \frac{\sin^3 \theta + \sin 3\theta}{\sin \theta} = 3$$

$$Q.33 \ 4(\sin^6 A + \cos^6 A) = 4 - 3\sin^2 2A = 1 + 3\cos^2 2A$$

$$Q.34 \ \cos^6 \theta - \sin^6 \theta = 1 - \frac{1}{4} \sin^2 2\theta$$

$$Q.35 \ \frac{\sin^3 \theta + \sin 3\theta}{\cos^3 \theta \cos 3\theta} = \cot \theta$$

$$Q.36 \ 1 - \tan 20^\circ \cdot \tan 25^\circ = \tan 20^\circ + \tan 25^\circ$$

$$Q.37 \ \tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$$

$$Q.38 \ 1 - \tan 30^\circ \cdot \tan 15^\circ = \tan 30^\circ + \tan 15^\circ$$

$$Q.39 \ \frac{1}{\sin A} - \frac{1}{\cos A} = \cot A$$

$$\tan 2A - \tan A \quad \cot 2A - \cot A$$

$$\text{Q.40 } \frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A} = \cot 2A$$

$$\text{Q.41 } \frac{1}{\tan 3A + \tan A} - \frac{1}{\cot 3A + \cot A} = \cot 4A$$

$$\text{Q.42 } 4\cos^3\theta\sin 3\theta + 4\sin^3\theta.\cos 3\theta = 3\sin 4\theta$$

$$\text{Q.43 } 1 + 8\cos^4 A - 8\cos^2 A = \cos 4A$$

$$\text{Q.44 } \sqrt{2 + \sqrt{2 + 2\cos 4A}} = 2\cos 2A$$

$$\text{Q.45 } \text{If } \tan A = b/a, \text{ prove that } a\cos 2\theta + b\sin 2\theta = a$$

$$\text{Q.46 } \text{If } \cos A = \frac{1}{2}(a + 1/a), \text{ prove that:}$$

$$\text{(a) } \cos 2A = \frac{1}{2}(a^2 + 1/a^2)$$

$$\text{(b) } \cos 3A = \frac{1}{2}(a^3 + 1/a^3)$$

$$\text{Q.47 } \text{If } \tan A = 1/7 \text{ and } \tan B = 1/3, \text{ prove that: } \cos 2A = \sin 4B$$

$$\text{Q.48 } \text{If } \frac{\tan A}{\tan B} = \frac{3}{4}, \text{ prove that:}$$

$$\text{(a) } \tan(A + B) = \frac{5\sin 2B}{5\cos 2B - 1}$$

$$\text{(b) } \tan(A - B) = \frac{\sin 2B}{5 - \cos 2B}$$

$$\text{Q.49 } \operatorname{cosec} 2A + \cot 4A = \cot A - \operatorname{cosec} 4A$$

$$\text{Q.50 } \cos^8\theta + \sin^8\theta = 1 - \sin^2 2\theta + 1/8 \sin^4 2\theta$$

$$\text{Q.51 } \sin^3 A.\cos 3A + \cos^3 A.\sin 3A = \frac{3}{4} \sin 4A$$

$$\text{Q.52 } \tan(60^\circ + A).\tan(60^\circ - A) = \frac{2\cos 2A + 1}{2\cos 2A - 1}$$

### Exercise 4.3

#### Short Questions

$$\text{Q.1 } \text{If } \cos 30^\circ = \sqrt{3}/2, \text{ find the values of:}$$

$$\text{(a) } \sin 15^\circ \quad \text{(b) } \cos 15^\circ \quad \text{(c) } \tan 15^\circ$$

Q.2 If  $\cos 45^\circ = 1/\sqrt{2}$ , find the values of:

(a)  $\sin 22\frac{1}{2}^\circ$       (b)  $\cos 22\frac{1}{2}^\circ$       (c)  $\tan 22\frac{1}{2}^\circ$

Q.3 if  $\tan \theta/2 = 3/4$ , find the value of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ .

Q.4 If  $\tan A/2 = 1/2$ , find the value of  $\sin A$ ,  $\cos A$  and  $\tan A$ .

Q.5 If  $\sin \theta/2 = 3/5$ , find the values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ .

Q.6 If  $\cos \theta/2 = 3/5$  find the values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ .

Q.7 If  $\tan \theta/2 = 5/12$ , find the value of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ .

Q.8 If  $2\cos 330^\circ = \sqrt{3}$ , find the value of  $\cos 165^\circ$ .

Q.9 If  $\sin \alpha/3 = 1/2$ , prove that:  $\sin \alpha = 1$ .

Q.10 If  $\sin \theta/3 = 4/5$ , prove that:  $\sin \theta = 44/125$

Q.11 If  $\sin \alpha/3 = 3/5$ , prove that  $\sin \alpha = 117/125$

Q.12 If  $\cos \theta/3 = 2/3$ , prove that:  $\cos \theta = -22/27$

Q.13 If  $\tan A/3 = 1/5$ , prove that:  $\tan A = 37/55$

Q.14 If  $\cos \alpha/2 = 1/2$ , prove that:  $\cos \alpha = -1/2$

Prove the following relations

Q.15  $\frac{1 - \cos A}{\sin A} = \tan A/2$

Q.16  $\frac{\sin A}{1 + \cos A} = \tan A/2$

Q.17  $\frac{1 + \cos A}{\sin A} = \cot A/2$

Q.18  $\frac{\sin A}{1 - \cos A} = \cot A/2$

Q.19  $2\sin^2 \theta/2 \cdot \cot \theta/2 = \sin \theta$

Q.20  $(\sin \theta/2 - \cos \theta/2)^2 = 1 - \sin \theta$

Q.21  $(\sin \theta/2 + \cos \theta/2)^2 = \sin \theta + 1$

$$\text{Q.22 } \frac{1 + \cos\alpha}{1 - \cos\alpha} = \cot^2\alpha/2$$

$$\text{Q.23 } \sin\theta/2 \cdot \cos^3\theta/2 + \cos\theta/2 \cdot \sin^3\theta/2 = 1/2\sin\theta$$

$$\text{Q.24 } \frac{2\tan\theta/2}{1 + \tan^2\theta/2} = \sin\theta$$

$$\text{Q.25 } \frac{1 - \tan^2\theta/2}{1 + \tan^2\theta/2} = \cos\theta$$

$$\text{Q.26 } \frac{\sin 2A}{2\sin A} = \cos^2 A/2 - \sin^2 A/2$$

$$\text{Q.27 } \frac{\cos^3\theta/2 + \sin^3\theta/2}{\cos\theta/2 + \sin\theta/2} = 1 - 1/2\sin\theta$$

$$\text{Q.28 } \sqrt{1 + \sin\theta} = \cos\theta/2 + \sin\theta/2$$

$$\text{Q.29 } \frac{\sec A + 1}{\sec A} = 2\cos^2 A/2$$

$$\text{Q.30 } \cos^4 A/2 - \sin^4 A/2 = \cos A$$

$$\text{Q.31 } \frac{\sin\theta + \sin\theta/2}{1 + \cos\theta + \cos\theta/2} = \tan\theta/2$$

$$\text{Q.32 } 2\sin(\pi^c/4 - \theta) \sin(3\pi^c/4 + \theta) = 1 \sin 2\theta$$

### Long Questions

$$\text{Q.1 } \frac{\sin A/2 + \sin A}{1 + \cos A/2 + \cos A} = \tan A/2$$

$$\text{Q.2 } \frac{\sin 2\alpha}{1 + \cos\alpha} - \frac{\cos\alpha}{1 + \cos\alpha} = \tan\alpha/2$$

$$\text{Q.3 } \frac{2\sin\beta + \sin 2\beta}{2\sin\beta - \sin 2\beta} = \cot^2\beta/2$$

$$\text{Q.4 } \frac{2\sin A - \sin 2A}{2\sin A + \sin 2A} = \tan 2A/2$$

$$\text{Q.5 } \frac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta} = \cot\theta/2$$

$$\text{Q.6 } \frac{\cot A/2 + \tan A/2}{\cot A/2 - \tan A/2} = \sec A$$

$$\text{Q.7 } \frac{\cos A/2 - \sin A/2}{\cos A/2 + \sin A/2} = \sec A - \tan A$$

$$\text{Q.8 } \frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \theta/2$$

$$\text{Q.9 } \tan(\pi/4 + A/2) = \sec A + \tan A$$

$$\text{Q.10 } \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \tan(45^\circ + A/2)$$

$$\text{Q.11 } \sqrt{\frac{1 - \sin A}{1 + \sin A}} = \tan(45^\circ - A/2)$$

$$\text{Q.12 } \sec(45^\circ + A/2) \cdot \sec(45^\circ - A/2) = \sec A$$

$$\text{Q.13 } \tan(\pi/4 - A/2) = \frac{\cos A}{1 + \sin A}$$

$$\text{Q.14 } \cot(A/2 + 45^\circ) - \tan(A/2 - 45^\circ) = 2(\sec A - \tan A)$$

$$\text{Q.15 } \tan(\pi/4 + \theta/2) + \tan(\pi/4 - \theta/2) = 2\sec \theta$$

$$\text{Q.16 } 1 - 2\sin^2(\pi/4 - \theta/2) = \sin \theta$$

$$\text{Q.17 } \frac{2\tan(\pi/4 - \theta/2)}{1 + \tan^2(\pi/4 - \theta/2)} = \cos \theta$$

$$\text{Q.18 } \cot A/2 - \tan A/2 = 2\cot A$$

$$\text{Q.19 } \frac{1 - \tan^2(\pi/4 - \theta/4)}{1 + \tan^2(\pi/4 - \theta/4)} = \sin \theta/2$$

$$\text{Q.20 } (\cos A - \cos B)^2 + (\sin A - \sin B)^2 = 4\sin^2(A - B)/2$$

$$\text{Q.21 } \tan \theta \cdot \tan(\pi/3 + \theta) \cdot \tan(\pi/3 - \theta) = \tan 3\theta$$

$$\text{Q.22 } \frac{\tan(\pi/4 + A) + \tan(\pi/4 - A)}{\tan(\pi/4 + A) - \tan(\pi/4 - A)} = \operatorname{cosec} 2A$$

$$\text{Q.23 } \text{If } \sin \alpha = \frac{1}{2}(a + 1/a), \text{ prove that: } \sin \alpha = \frac{1}{2}(a^2 + 1/a^2)$$

$$\text{Q.24 } \text{If } \cos \alpha/3 = \frac{1}{2}(a + 1/a), \text{ prove that: } \cos \alpha = \frac{1}{2}(a^3 + 1/a^3)$$

Q.25 Prove that  $\cot 22\frac{1}{2}^\circ - \tan 22\frac{1}{2}^\circ = 2$

Q.26 Prove that:  $\sec(\pi/4 + \theta/2) \cdot \sec(\pi/4 - \theta/2) = 2\sec\theta$

Q.27 Prove that:  $\frac{\sin A + 2\sin 3A - \sin 7A}{2\cos A - \cos 3A - \cos 5A} = \cos 2A + \cot A \cdot \sin 2A$

#### Exercise 4.4

#### Short Questions

(a) Express the following products into sum or difference:

- (i)  $2\sin 4\theta \cdot \cos 3\theta$
- (ii)  $2\cos 4\theta \cdot \sin 3\theta$
- (iii)  $\cos 4\theta \cdot \cos 3\theta$
- (iv)  $\sin 4\theta \cdot \sin 3\theta$
- (v)  $2\sin 50^\circ \cdot \cos 32^\circ$
- (vi)  $\cos 72^\circ \cdot \sin 43^\circ$
- (vii)  $\cos 27^\circ \cdot \cos 15^\circ$
- (viii)  $2\cos 61^\circ \cdot \cos 39^\circ$
- (ix)  $\sin 42^\circ \cdot \sin 16^\circ$
- (x)  $\sin 61^\circ \cdot \sin 39^\circ$
- (xi)  $\sin 36^\circ \cdot \sin 24^\circ$
- (xii)  $\sin 51^\circ \cdot \sin 10^\circ$
- (xiii)  $\cos 22^\circ \cdot \sin 50^\circ$
- (xiv)  $\cos 140^\circ \cdot \sin 40^\circ$
- (xv)  $2\sin 5x \cdot \cos 2x$
- (xvi)  $2\sin 2x \cdot \cos x$
- (xvii)  $\sin 6x \cdot \cos 4x$
- (xviii)  $2\cos 3\theta \cdot \sin 5\theta$
- (xix)  $\sin 33^\circ \cdot \sin 25^\circ$
- (xx)  $\cos 27^\circ \cdot \cos 21^\circ$
- (xxi)  $\cos 105^\circ \cdot \cos 15^\circ$
- (xxii)  $\sin 105^\circ \cdot \sin 15^\circ$
- (xxiii)  $\sin 73^\circ \cdot \sin 15^\circ$
- (xxiv)  $\cos 15^\circ \cdot \cos 75^\circ$

(b) Express the following sum or difference into product:

- (i)  $\sin 5\theta + \sin 3\theta$
- (ii)  $\cos 5\theta + \cos 3\theta$
- (iii)  $\sin 5\theta - \sin 3\theta$
- (iv)  $\cos 5\theta - \cos 3\theta$

- (v)  $\sin 50^\circ + \sin 30^\circ$
- (vi)  $\sin 50^\circ + \sin 70^\circ$
- (vii)  $\cos 70^\circ - \cos 40^\circ$
- (viii)  $\cos 70^\circ + \cos 40^\circ$
- (ix)  $\cos 50^\circ - \cos 40^\circ$
- (x)  $\sin 100^\circ - \sin 50^\circ$
- (xi)  $\sin 105^\circ + \sin 140^\circ$
- (xii)  $\sin 46^\circ - \sin 20^\circ$
- (xiii)  $\sin 84^\circ - \sin 116^\circ$
- (xiv)  $\cos 46^\circ - \cos 20^\circ$
- (xv)  $\cos 48^\circ - \cos 20^\circ$
- (xvi)  $\cos 110^\circ + \cos 130^\circ$
- (xvii)  $\sin 5\theta - \sin 7\theta$
- (xviii)  $\cos 32^\circ - \cos 16^\circ$
- (xix)  $\sin(\alpha + 3\beta) - \sin(3\alpha + \beta)$
- (xx)  $\cos(x + 2y) - \cos(2x + y)$

(c) Prove the following relations:

Q.1  $\sin(x + y) + \sin(x - y) = 2\sin x \cdot \cos y$

Q.2  $\sin(x + y) - \sin(x - y) = 2\cos x \cdot \sin y$

Q.3  $\cos(x + y) + \cos(x - y) = 2\cos x \cdot \cos y$

Q.4  $\cos(x + y) - \cos(x - y) = -2\sin x \cdot \sin y$

Q.5  $\sin 6\theta + \sin 2\theta = 2\sin 4\theta \cdot \cos 2\theta$

Q.6  $\sin 5A + \sin 7A = 2\sin 6A \cdot \cos A$

Q.7  $\cos 7A + \cos A = 2\cos 4A \cdot \cos 3A$

Q.8  $\sin 4\theta - \sin 2\theta = 2\sin \theta \cdot \cos 3\theta$

Q.9  $\cos \theta - 2\cos 3\theta = 2\sin^2 \frac{\theta}{2} \cdot \sin^3 \frac{\theta}{2}$

Q.10  $\sin 40^\circ + \sin 10^\circ = 2\cos 65^\circ \cdot \sin 75^\circ$

Q.11  $\cos 32^\circ - \cos 16^\circ = -2\sin 24^\circ \cdot \sin 8^\circ$

Q.12  $2\sin 40^\circ \cdot \cos 20^\circ = \sin 60^\circ + \sin 20^\circ$

Q.13  $2\cos 75^\circ \cdot \sin 25^\circ = \sin 100^\circ - \sin 50^\circ$

$$Q.14 \ 2\cos 120^\circ \cdot \cos 10^\circ = \cos 110^\circ + \cos 130^\circ$$

$$Q.15 \ 2\sin 125^\circ \cdot \sin 15^\circ = \cos 130^\circ - \cos 120^\circ$$

$$Q.16 \ \frac{\sin A + \sin B}{\cos A + \cos B} = \tan^{(A+B)/2}$$

$$Q.17 \ \frac{\sin 7A + \sin 3A}{\cos 7A + \cos 3A} = \tan 5A$$

$$Q.18 \ \frac{\sin 3A - \sin A}{\cos A - \cos 3A} = \cot 2A$$

$$Q.19 \ \frac{\cos 2x - \cos 2y}{\sin 2x + \sin 2y} = \tan(y - x)$$

$$Q.20 \ \frac{\sin A - \sin B}{\cos B - \cos A} = \cot^{(A+B)/2}$$

$$Q.21 \ \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

$$Q.22 \ \sin A + \sin B = \tan^{(A+B)/2} \cdot \cot^{(A-B)/2}$$

$$Q.23 \ \cos B - \cos A = \tan^{(A+B)/2} \cdot \cot^{(A-B)/2}$$

$$Q.24 \ \sin 2A + \sin 2B = \tan(A+B) \cdot \cot(A-B)$$

$$Q.25 \ \sin 75^\circ + \sin 15^\circ = \frac{1}{2}$$

$$Q.26 \ \sin 75^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$$

$$Q.27 \ \cos 50^\circ + \cos 70^\circ = \cos 10^\circ$$

$$Q.28 \ \sin 50^\circ - \sin 70^\circ = -\sin 10^\circ$$

$$Q.29 \ \sin 50^\circ + \sin 10^\circ = \sin 70^\circ$$

$$Q.30 \ \sin 50^\circ + \sin 70^\circ = \sqrt{3} \cos 10^\circ$$

$$Q.31 \ \sin 10^\circ - \sin 50^\circ = -\sqrt{3} \sin 20^\circ$$

$$Q.32 \ 2\cos(45^\circ + A) \cdot \cos(45^\circ - A) = \cos 2A$$

Long Questions:

Prove the following relations

$$\text{Q.1 } \cos 40^\circ - \sin 40^\circ = \sqrt{2} \sin 15^\circ$$

$$\text{Q.2 } \cos 40^\circ + \sin 40^\circ = \sqrt{2} \cos 5^\circ$$

$$\text{Q.3 } \sin 10^\circ + \cos 40^\circ = \sin 70^\circ$$

$$\text{Q.4 } \cos 10^\circ + \cos 110^\circ + \cos 130^\circ = 0$$

$$\text{Q.5 } \cos 30^\circ + \sin 50^\circ = \sqrt{2} \cos 5^\circ$$

$$\text{Q.6 } \sin 70^\circ - \sin 50^\circ - \sin 10^\circ = 0$$

$$\text{Q.7 } \frac{\cos 10^\circ - \sin 10^\circ}{\cos 10^\circ + \sin 10^\circ} = \tan 35^\circ$$

$$\text{Q.8 } \frac{\cos 75^\circ + \cos 15^\circ}{\cos 75^\circ - \sin 15^\circ} = \sqrt{3}$$

$$\text{Q.9 } \frac{\cos 40^\circ - \sin 40^\circ}{\cos 40^\circ + \sin 40^\circ} = \tan 5^\circ$$

$$\text{Q.10 } \frac{\cos 14^\circ - \sin 14^\circ}{\cos 14^\circ + \sin 14^\circ} = \cot 59^\circ$$

$$\text{Q.11 } \frac{\sin 2A + \sin 5A - \sin A}{\cos 2A + \cos 5A + \cos A} = \tan 2A$$

$$\text{Q.12 } \frac{\cos A - \cos 2A + \cos 3A}{\sin A - \sin 2A + \sin 3A} = \cot 2A$$

$$\text{Q.13 } \frac{\sin x + \sin 2x + \sin 4x + \sin 5x}{\cos x + \cos 2x + \cos 4x + \cos 5x} = \tan 3x$$

$$\text{Q.14 } \frac{\sin 2\theta + \sin 3\theta + \sin 5\theta + \sin 6\theta}{\cos 2\theta + \cos 3\theta + \cos 5\theta + \cos 6\theta} = \tan 4\theta$$

$$\text{Q.15 } \frac{\sin 7A - \sin 5A - \sin 3A + \sin A}{\cos 7A + \cos 3A - \cos 5A - \cos A} = \tan 2A$$

$$\text{Q.16 } \frac{(\cos \theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin \theta)(\cos 4\theta - \cos 6\theta)} = \tan 2\theta$$

$$\text{Q.17 } \frac{\sin 8\theta \cos \theta - \sin 6\theta \cdot \cos 3\theta}{\sin 2\theta \cdot \cos \theta - \sin 3\theta \cdot \sin 4\theta} = \tan 2\theta$$

$$\text{Q.18 } \frac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B} = \tan(A + B)$$

$$\text{Q.19 } \sin(45^\circ - \theta) \cdot \sin(45^\circ + \theta) = \frac{1}{2} \cos 2\theta$$

$$\text{Q.20 } \cos \alpha + \cos\left(\frac{2\pi}{3} - \alpha\right) + \cos\left(\frac{2\pi}{3} + \alpha\right) = 0$$

$$\text{Q.21 } \sin \theta + \sin\left(\frac{2\pi}{3} + \theta\right) + \sin\left(\frac{4\pi}{3} + \theta\right) = 0$$

$$\text{Q.22 } 2\sin(45^\circ + A) \cdot \sin(45^\circ - A) = \cos 2A$$

$$\text{Q.23 } \sin(150^\circ + x) + \sin(150^\circ - x) = \cos x$$

$$\text{Q.24 } \cos(45^\circ + x) + \cos(45^\circ - x) = \sqrt{2} \cos x$$

$$\text{Q.25 } \sin 5A - \sin 3A + 2\sin A \cdot \cos 2A = 2\sin 2A \cdot \cos 3A$$

$$\text{Q.26 } \sin 4\theta \cdot \cos 2\theta + \cos 3\theta \cdot \sin \theta = \sin 5\theta \cdot \cos \theta$$

$$\text{Q.27 } \cos 7A + \cos 5A + \cos 3A + \cos A = 4\cos A \cdot \cos 2A \cdot \cos 3A$$

$$\text{Q.28 } (\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2 \frac{(A+B)}{2}$$

$$\text{Q.29 } (\sin A - \sin B)^2 + (\cos A - \cos B)^2 = 4\sin^2 \frac{(A-B)}{2}$$

$$\text{Q.30 } \cos A + \cos B + \cos C + \cos(A + B + C) = 4\cos \frac{(A+B)}{2} \cdot \cos \frac{(B+C)}{2} \cdot \cos \frac{(C+A)}{2}$$

$$\text{Q.31 } 8\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ = \sqrt{3}$$

$$\text{Q.32 } \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ = \frac{1}{8}$$

$$\text{Q.33 } \cos 40^\circ \cdot \cos 100^\circ \cdot \cos 160^\circ = \frac{1}{8}$$

$$\text{Q.34 } 16\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = 1$$

$$\text{Q.35 } \sin 20^\circ \cdot \sin 30^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ = \frac{\sqrt{3}}{16}$$

$$\text{Q.36 } 16\cos 20^\circ \cdot \cos 30^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ = \sqrt{3}$$

$$\text{Q.37 } \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \frac{3}{16}$$

$$\text{Q.38 } \tan 6^\circ \cdot \tan 42^\circ \cdot \tan 66^\circ \cdot \tan 78^\circ = 1$$

$$\text{Q.39 } \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 80^\circ = \sqrt{3}$$

Q.40  $\cos 10^\circ \cdot \cos 30^\circ \cdot \cos 130^\circ \cdot \cos 110^\circ = 3/16$

Q.41  $\cos 40^\circ \cdot \cos 80^\circ \cdot \cos 160^\circ = -1/8$

Q.42 If  $\cos A + \cos B = 1/2$  and  $\sin A + \sin B = 1/4$  prove that:  
(a)  $\tan^{(A+B)/2} = 1/2$       (b)  $\cos(A + B) = 3/5$

Q.43 If  $\frac{\sin A}{\sin B} = k$  prove that:  $\frac{\tan^{(A-B)/2}}{\tan^{(A+B)/2}} = \frac{k-1}{k+1}$

Q.44 If  $\frac{\sin(A-B)}{\sin(A+B)} = k$  prove that:  $(k-1) \tan A = (k+1) \tan B$

Q.45 If  $\cos \alpha + \cos \beta = 1/3$  and  $\sin \alpha + \sin \beta = 1/4$  prove that:  $\tan^{(\alpha+\beta)/2} = 3/4$

#### Exercise 4.5

#### Long Questions

(a) If  $A + B + C = \pi$

Q.1  $\sin A + \sin B + \sin C = 4 \cos^{A/2} \cdot \cos^{B/2} \cdot \cos^{C/2}$

Q.2  $\cos A + \cos B + \cos C = 1 + 4 \sin^{A/2} \cdot \sin^{B/2} \cdot \sin^{C/2}$

Q.3  $\cos A - \cos B + \cos C = -1 + 4 \cos^{A/2} \cdot \cos^{B/2} \cdot \cos^{C/2}$

Q.4  $\cos A + \cos B - \cos C = -1 + 4 \cos^{A/2} \cdot \cos^{B/2} \cdot \cos^{C/2}$

Q.5  $-\cos A + \cos B + \cos C = -1 + 4 \sin^{A/2} \cdot \sin^{B/2} \cdot \sin^{C/2}$

Q.6  $\sin A - \sin B + \sin C = 4 \sin^{A/2} \cdot \sin^{B/2} \cdot \sin^{C/2}$

Q.7  $\sin A + \sin B - \sin C = 4 \sin^{A/2} \cdot \sin^{B/2} \cdot \sin^{C/2}$

Q.8  $-\sin A + \sin B + \sin C = 4 \cos^{A/2} \cdot \sin^{B/2} \cdot \sin^{C/2}$

Q.9  $\tan A + \tan B = \tan A \cdot \tan B \cdot \tan C$

Q.10  $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A = 1$

Q.11  $\tan A/2 \cdot \tan B/2 + \tan B/2 \cdot \tan C/2 + \tan C/2 \cdot \tan A/2 = 1$

Q.12  $\cot A/2 + \cot B/2 + \cot C/2 = 4 \cot^{A/2} \cdot \cot^{B/2} \cdot \cot^{C/2}$

Q.13  $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cdot \cos B \cdot \cos C$

$$\text{Q.14 } \sin 2A - \sin 2B + \sin 2C = 4\cos A \cdot \sin B \cdot \cos C$$

$$\text{Q.15 } -\sin 2A + \sin 2B + \sin 2C = 4\sin A \cdot \cos B \cdot \cos C$$

$$\text{Q.16 } \cos 2A + \cos 2B + \cos 2C = 1 - 4\cos A \cdot \cos B \cdot \cos C$$

$$\text{Q.17 } \cos 2A - \cos 2B + \cos 2C = 1 - 4\sin A \cdot \cos B \cdot \sin C$$

$$\text{Q.18 } \cos 2A + \cos 2B - \cos 2C = 1 - 4\sin A \cdot \sin B \cdot \cos C$$

$$\text{Q.19 } -\cos 2A + \cos 2B + \cos 2C = 1 - 4\cos A \cdot \sin B \cdot \sin C$$

$$\text{Q.20 } \cos 2A - \cos 2B - \cos 2C = 4\cos A \cdot \sin B \cdot \sin C - 1$$

$$\text{Q.21 } \frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = 8\sin^{A/2} \cdot \sin^{B/2} \cdot \sin^{C/2}$$

$$\text{Q.22 } \frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = 8\cos^{A/2} \cdot \cos^{B/2} \cdot \sin^{C/2}$$

$$\text{Q.23 } \frac{\sin 2A + \sin 2B + \sin 2C}{\sin A - \sin B + \sin C} = 8\cos^{A/2} \cdot \sin^{B/2} \cdot \cos^{C/2}$$

$$\text{Q.24 } \frac{\sin 2A + \sin 2B + \sin 2C}{-\sin A + \sin B + \sin C} = 8\sin^{A/2} \cdot \cos^{B/2} \cdot \sin^{C/2}$$

$$\text{Q.25 } \tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot \tan 2B \cdot \tan 2C$$

$$\text{Q.26 } \frac{\sin B + \sin C - \sin A}{\sin A + \sin B + \sin C} = \tan^{B/2} \cdot \tan^{C/2}$$

$$\text{Q.27 } \frac{\sin A + \sin B + \sin C}{\sin A - \sin B + \sin C} = \tan^{A/2} \cdot \tan^{C/2}$$

$$\text{Q.28 } \frac{\sin A + \sin B + \sin C}{\sin A + \sin B - \sin C} = \tan^{A/2} \cdot \tan^{B/2}$$

$$\text{Q.29 } \cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cdot \cos B \cdot \cos C$$

$$\text{Q.30 } \sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cdot \cos B \cdot \cos C$$

$$\text{Q.31 } \sin^2 A - \sin^2 B + \sin^2 C = 2\sin A \cdot \cos B \cdot \sin C$$

$$\text{Q.32 } \sin^2 A + \sin^2 B - \sin^2 C = 2\sin A \cdot \sin B \cdot \sin C$$

$$Q.33 -\sin^2 A + \sin^2 B + \sin^2 C = 2\cos A \cdot \sin B \cdot \sin C$$

$$Q.34 \cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A \cdot \sin B \cdot \cos C$$

$$Q.35 \cos^2 A - \cos^2 B + \cos^2 C = 1 - 2\sin A \cdot \cos B \cdot \sin C$$

$$Q.36 -\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cdot \sin B \cdot \sin C$$

$$Q.37 \cos^2 A + \cos^2 B - \sin^2 C = -2\cos A \cdot \cos B \cdot \cos C$$

$$Q.38 \sin^2 A/2 + \sin^2 B/2 + \sin^2 C/2 = 1 - 2\sin A/2 \cdot \sin B/2 \cdot \sin C/2$$

$$Q.39 \sin^2 A/2 + \sin^2 B/2 - \sin^2 C/2 = 1 - 2\cos A/2 \cdot \cos B/2 \cdot \sin C/2$$

$$Q.40 \sin^2 A/2 - \sin^2 B/2 + \sin^2 C/2 = 1 - 2\cos A/2 \cdot \sin B/2 \cdot \cos C/2$$

$$Q.41 -\sin^2 A/2 + \sin^2 B/2 + \sin^2 C/2 = 1 - 2\sin A/2 \cdot \cos B/2 \cdot \cos C/2$$

$$Q.42 \cos^2 A/2 + \cos^2 B/2 + \cos^2 C/2 = 2 + 2\sin A/2 \cdot \sin B/2 \cdot \sin C/2$$

$$Q.43 \cos^2 A/2 + \cos^2 B/2 - \cos^2 C/2 = 2\cos A/2 \cdot \cos B/2 \cdot \sin C/2$$

$$Q.44 \cos^2 A/2 - \cos^2 B/2 + \cos^2 C/2 = 2\cos A/2 \cdot \sin B/2 \cdot \cos C/2$$

$$Q.45 -\cos^2 A/2 - \cos^2 B/2 + \cos^2 C/2 = 2\sin A/2 \cdot \cos B/2 \cdot \cos C/2$$

$$Q.46 \sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C) = 4\sin A \cdot \sin B \cdot \sin C$$

$$Q.47 \frac{\cos A}{\sin B \cdot \sin C} + \frac{\cos B}{\sin C \cdot \sin A} + \frac{\cos C}{\sin A \cdot \sin B} = 2$$

$$Q.48 \frac{\sin A}{\cos B \cdot \cos C} + \frac{\sin B}{\cos C \cdot \cos A} + \frac{\sin C}{\cos A \cdot \cos B} = 2$$

$$Q.49 \sin A \cdot \sin B \cdot \cos C + \sin A \cdot \sin C \cdot \cos B + \sin B \cdot \sin C \cdot \cos A = 1 + \cos A \cdot \cos B \cdot \cos C$$

$$Q.50 \text{ If } \alpha + \beta + \gamma = \pi/2 \text{ prove that } \sin 2\alpha + \sin 2\beta + \sin 2\gamma + 2\sin \alpha \cdot \sin \beta \cdot \sin \gamma = 1$$

$$Q.51 \text{ If } A + B + C = \pi/2 \text{ prove that } \tan A \cdot \tan B + \tan B \cdot \tan C + \tan C \cdot \tan A = 1$$

$$Q.52 \text{ If } A + B + C = \pi, \text{ prove that:}$$

$$A + 4\sin^{(\pi-A)/4} \cdot \sin^{(\pi-B)/4} \cdot \sin^{(\pi-C)/4} = \sin^A/2 + \sin^B/2 + \sin^C/2$$

$$Q.53 \text{ If } A + B + C = \pi, \text{ prove that}$$

$$1 + 4\sin^{(B+C)/4} \cdot \sin^{(C+A)/4} \cdot \sin^{(A+B)/4} = \sin^A/2 + \sin^B/2 + \sin^C/2$$

Q.54 If  $A + B + C = \pi$ , prove that

$$\cos A + \cos B + \cos C = 4 \cos \frac{(\pi-A)}{4} \cdot \cos \frac{(\pi-B)}{4} \cdot \cos \frac{(\pi-C)}{4}$$

Q.55 If  $A + B + C = \pi$ , prove that

$$\sin A \cos B \cos C + \sin B \cos C \cos A + \sin C \cos A \cos B = \sin A \sin B \sin C$$

Q.56 If  $A + B + C = \pi$ , prove that

$$\sin(B + C - A) + \sin(C + A - B) - \sin(A + B - C) = 4 \cos A \cos B \sin C$$

Q.57 If  $A + B + C = \pi$ , prove that:

$$\cos(B + C - A) + \cos(C + A - B) + \cos(A + B - C) = 1 + 4 \cos A \cos B \cos C$$

Q.58 If  $A + B + C = \pi^c$ , prove that:

$$\sin A \sin B \sin C (\cot A + \cot B + \cot C) = 1 + \cos A \cos B \cos C$$

Q.59 If  $P + Q + R = \left(\frac{3\pi}{2}\right)^c$ , prove that:

$$\sin P \cdot \cos Q \cdot \sin R + \cos P \cdot \sin Q \cdot \sin R + \sin P \cdot \sin Q \cdot \cos R = \cos P \cdot \cos Q \cdot \cos R$$

## Exercise 4.6

### Short Questions

Q.1 Solve:  $\sin \theta = \frac{\sqrt{3}}{2}$   $(0 \leq \theta \leq 360^\circ)$

Q.2 Solve:  $2 \sin \theta = 1$   $(0 \leq \theta \leq 360^\circ)$

Q.3 Solve:  $\cos \theta = \frac{\sqrt{3}}{2}$   $(0 \leq \theta \leq 360^\circ)$

Q.4 Solve:  $\cos \theta = \frac{1}{2}$   $(0 \leq \theta \leq 360^\circ)$

Q.5 Solve:  $\cos \theta = -\frac{1}{\sqrt{2}}$   $(0 \leq \theta \leq 360^\circ)$

Q.6 Solve:  $2 \sin \theta + \sqrt{3} = 0$   $(0 \leq \theta \leq 360^\circ)$

Q.7 Solve:  $2 \cos \theta + 1 = 0$   $(0 \leq \theta \leq 360^\circ)$

Q.8 Solve:  $\sqrt{2} \sec \theta = 0$   $(0 \leq \theta \leq 360^\circ)$

Q.9 Solve:  $2 \cos \theta - 1 = 0$   $(0 \leq \theta \leq 360^\circ)$

Q.10 Solve:  $\tan \theta = \sqrt{3}$   $(0 \leq \theta \leq 360^\circ)$

- Q.11 Solve:  $\tan\theta = -1$   $(0 \leq \theta \leq 360^\circ)$
- Q.12 Solve:  $\sqrt{3} \tan\theta + 1 = 0$   $(0 \leq \theta \leq 360^\circ)$
- Q.13 Solve:  $\tan\theta + \sqrt{3} = 0$   $(0 \leq \theta \leq 360^\circ)$
- Q.14 Solve:  $\operatorname{cosec}\theta = 2$   $(0 \leq \theta \leq 360^\circ)$
- Q.15 Solve:  $\cot\theta = 1$   $(0 \leq \theta \leq 180^\circ)$
- Q.16 Solve:  $\sec\theta - 1 =$   $(0 \leq \theta \leq 360^\circ)$
- Q.17 Solve:  $\sqrt{3} \cot\theta + 1 = 0$   $(0 \leq \theta \leq 360^\circ)$
- Q.18 Solve:  $3\sec^2\theta - 4 = 0$   $(0 \leq \theta \leq 360^\circ)$
- Q.19 Solve:  $\sin\theta \cdot \cot\theta - 1 = 0$   $(0 \leq \theta \leq 360^\circ)$
- Q.20 Solve:  $4\sin\theta - 3 = 0$   $(0 \leq \theta \leq 360^\circ)$
- Q.21 Solve:  $2\cos\theta = 1$   $(0 \leq \theta \leq 360^\circ)$
- Q.22 Solve:  $2\cos^2\theta = -\sqrt{3} \cos\theta$   $(0 \leq \theta \leq 360^\circ)$
- Q.23 Solve:  $\cos^2\theta/2 - \cos\theta/2 + 1/4 = 0$   $(0 \leq \theta \leq 180^\circ)$
- Q.24 Solve:  $\sin\theta - \tan\theta = 0$   $(0 \leq \theta \leq 90^\circ)$
- Q.25 Solve:  $2\sin^2\theta = \sqrt{3} \sin\theta$   $(0 \leq \theta \leq 90^\circ)$
- Q.26 Solve:  $\cos^2\theta = \sin^2\theta$   $(0 \leq \theta \leq 180^\circ)$
- Q.27 Solve:  $\sqrt{2}\sec^2\theta + 2 = 0$   $(0 \leq \theta \leq 360^\circ)$
- Q.28 Solve:  $\sin\theta = \cos\theta$ , find the value of  $\theta$ .  $(0 \leq \theta \leq 180^\circ)$
- Q.29 Solve:  $\cos 4x = \sin x$   $(0 \leq x \leq 90^\circ)$
- Q.30 If  $\sin\theta - \cos\theta = 0$ , find the acute value of  $\theta$ .

### Long Questions

- Q.1 Solve:  $\sqrt{3} \sin\alpha - \cos\alpha = \sqrt{2}$   $(0^\circ \leq \alpha \leq 180^\circ)$
- Q.2 Solve:  $2\cos^2\theta = 2\sin\theta$   $(0^\circ \leq \theta \leq 180^\circ)$

- Q.3 Solve:  $2\cos 2\theta = 2\sin\theta$   $(0^\circ \leq \theta \leq 180^\circ)$
- Q.4 Solve:  $1 + \cos\theta = 2\sin^2\theta$   $(0^\circ \leq \theta \leq 180^\circ)$
- Q.5 Solve:  $\sec\theta \cdot \tan\theta = \sqrt{2}$   $(0^\circ \leq \theta \leq 180^\circ)$
- Q.6 Solve:  $\cos^2\theta - \sin\theta = \frac{1}{4}$   $(0^\circ \leq \theta \leq 180^\circ)$
- Q.7 Solve:  $\sin 3\theta + \sin\theta = \sin 2\theta$   $(0^\circ \leq \theta \leq 180^\circ)$
- Q.8 Solve:  $2\cos^2\theta - \sin\theta = 2$   $(0^\circ \leq \theta \leq 180^\circ)$
- Q.9 Solve:  $\cos^2x = 3\sin^2x + 4\cos x$   $(0^\circ \leq x \leq 360^\circ)$
- Q.10 Solve for x, the equation  $\cos 3x + \cos x = 2\cos x$  where  $0^\circ \leq x \leq 360^\circ$ .
- Q.11 Solve:  $\cos \theta = \sqrt{3} \sin \theta - \sqrt{2}$   $(0^\circ \leq \theta \leq 360^\circ)$
- Q.12 Solve:  $\tan x + \cot x = 2$   $(0^\circ \leq x \leq 360^\circ)$
- Q.13 Solve:  $\tan x - \sin x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.14 Solve:  $2\sin x = \tan x$   $(0^\circ \leq x \leq 360^\circ)$
- Q.15 Solve:  $\operatorname{cosec}^2x - 2 = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.16 Solve:  $\operatorname{cosec} x - 2\sin x = 1$   $(0^\circ \leq x \leq 360^\circ)$
- Q.17 Solve:  $7\sin^2x + 3\cos^2x - 4 = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.18 Solve:  $2\cos x - \cot x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.19 Solve:  $\tan^2x - 3\sec x + 3 = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.20 Solve:  $2\cos^2x - 2\sin x = \frac{1}{2}$   $(0^\circ \leq x \leq 360^\circ)$
- Q.21 Solve:  $6\cos^2x = 1 - \cos x$   $(0^\circ \leq x \leq 360^\circ)$
- Q.22 Solve:  $4\sec^2x - 7\tan^2x = 3$   $(0^\circ \leq x \leq 360^\circ)$
- Q.23 Solve:  $\operatorname{cosec}^2x + \cot^2x = 3$   $(0^\circ \leq x \leq 360^\circ)$
- Q.24 Solve:  $\sin^2x + 2\cos^2x = 1$   $(0^\circ \leq x \leq 360^\circ)$
- Q.25 Solve:  $3\sin^2x + 4\cos x - 4 = 0$   $(0^\circ \leq x \leq 360^\circ)$

- Q.26 Solve:  $\sqrt{3}\cos^2x - \cosx = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.27 Solve:  $2\cos^2x - 5\cosx + 2 = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.28 Solve:  $6\cos^2x + 4\sin^2x = 5$   $(0^\circ \leq x \leq 360^\circ)$
- Q.29 Solve:  $4\cos^2x = 1$   $(0^\circ \leq x \leq 360^\circ)$
- Q.30 Solve:  $\sin 2x + \sin x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.31 Solve:  $\sin^2x - \cosx = 1$   $(0^\circ \leq x \leq 360^\circ)$
- Q.32 Solve:  $4\cosx + \secx - 4 = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.33 Solve:  $2\sinx + \cotx - \operatorname{cosec}x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.34 Solve:  $2\cos^2x + \sinx\cosx - \sin^2x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.35 Solve:  $\sin 2x - 4\sinx - \cosx + 2 = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.36 Solve:  $\sin 9x - \sinx = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.37 Solve:  $\tan 2x - \cotx - \operatorname{cosec}x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.38 Solve:  $\cosx + \cos 2x + \cos 3x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.39 Solve:  $\tanx + \tan 2x + \tan 3x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.40 Solve:  $\cot^2x - \operatorname{cosec}x - 1 = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.41 Solve:  $\cot^2x + \sinx - 1 = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.42 Solve:  $\sin^2\theta - \cos\theta = \frac{1}{4}$   $(0^\circ \leq \theta \leq 360^\circ)$
- Q.43 Solve:  $\sinx + \sin 2x + \sin 3x = 0$   $(0^\circ \leq x \leq 360^\circ)$
- Q.44 Solve:  $\cos^2\theta - \sin\theta = \frac{1}{4}$   $(0^\circ \leq \theta \leq 360^\circ)$
- Q.45 Solve:  $\sqrt{3}\cos\theta + \sin\theta = \sqrt{3}$   $(0^\circ \leq \theta \leq 360^\circ)$
- Q.46 Solve:  $\sqrt{3}\sinx + \cosx = 1$   $(0^\circ \leq x \leq 360^\circ)$
- Q.47 Solve:  $\sin\theta + \cos\theta = 1/\sqrt{2}$   $(0^\circ \leq \theta \leq 360^\circ)$
- Q.48 Solve:  $\sin\theta + \cos\theta = \sqrt{2}$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.49 Solve:  $\sqrt{3} \sin\theta + \cos\theta = 2$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.50 Solve:  $\sqrt{3}\cos\theta + \sin\theta = 1$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.51 Solve:  $\sin 2x - 1 = \cos x$   $(0^\circ \leq x \leq 360^\circ)$

Q.52 Solve:  $\sin\theta - \cos\theta = 1/\sqrt{2}$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.53 Solve:  $\sqrt{3}\sin\theta = \tan\theta$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.54 Solve:  $\tan\theta \cdot \sec\theta = \sqrt{2}$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.55 Solve:  $\sqrt{3} \sin\theta + \cos\theta = \sqrt{2}$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.56 Solve:  $\sin\theta + \sqrt{3}\cos\theta = \sqrt{2}$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.57 Solve:  $\sqrt{3} \sin\theta - \cos\theta = \sqrt{2}$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.58 Solve:  $\sqrt{3} \cos x + \sin x = \sqrt{3}$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.59 Solve:  $\sqrt{3} \cos\theta + \sin\theta = 2$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.60 Solve:  $\cos\theta - \sqrt{3} \sin\theta = -1$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.61 Solve:  $\sqrt{3} \cot^2\theta + 4\cot\theta = -\sqrt{3}$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.62 Solve:  $2\sin x \tan x + 1 = 2\sin x + \tan x$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.63 Solve:  $\tan^2\theta - (1 + \sqrt{3}) \tan\theta + \sqrt{3} = 0$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.64 Solve:  $\sin A = \sqrt{3}(1 - \cos A)$   $(0^\circ \leq A \leq 360^\circ)$

Q.65 If  $\tan x + \tan y = 2$  and  $2\cos x \cdot \cos y = 1$ , find the values of  $x$  and  $y$ .

Q.66 If  $\cot x + \cot y = 2$  and  $2\sin x \cdot \sin y = \sqrt{3}/2$ , find the values of  $x$  and  $y$ .

Q.67 If  $\tan x + \cot y = 2$  and  $2\cos x \cdot \sin y = 1/\sqrt{2}$ , find the values of  $x - y$ .

Q.68 Solve:  $\sqrt{3} \cot A = \sqrt{3}/\sin A - 1$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.69 Solve:  $\cot^2\theta + [\sqrt{3} + (1/\sqrt{3})] \cot \theta = -1$   $(0^\circ \leq \theta \leq 360^\circ)$

Q.70 Solve:  $[\sqrt{3}/\sin 2\theta] + [1/\cos 2\theta] = 4$   $(0^\circ \leq \theta \leq 360^\circ)$

#### Exercise 4.7

- Q.1 The angle of elevations on the top of a house from the two places on the same side are  $30^\circ$  and  $45^\circ$ . If the distance between two places is 20m, find the height of the house.
- Q.2 The angles of elevations on a balloon flying 200m high on the sky from the two places on the same side are  $65^\circ$  and  $55^\circ$ . Find the distance between the two places.
- Q.3 The angle of elevations on the top of a house from the two places on the same side are  $35^\circ$  and  $45^\circ$ . If the distance between two places is 30m, find the height of the house.
- Q.4 A pencil of 15 cm long and length of its shadow are same at noon. After two hours it was observed that the length of shadow is 25.98 cm. Find the altitude of the sun at two places.
- Q.5 A and B are two points on the opposite sides of a tower of 100m high on the same level of ground. If the angles of elevation of the top of the tower from the points A and B are  $60^\circ$  and  $22^\circ$  respectively, find the distance between two points.
- Q.6 The angle of elevation of the top of a tower 200m high from a point on east is  $60^\circ$  and that from on west is  $40^\circ$ . Find the distance between two points.
- Q.7 An angle of elevation on the top of a hill from a point on east is  $55^\circ$  and that from the point on west is  $35^\circ$ . If the distance between two points on the same plane is 1800m, find the height of the hill.
- Q.8 The angle of elevations on the top of a cliff from the two places on the opposite sides on the same plane are  $30^\circ$  and  $60^\circ$ . If the height of the cliff is 80m, find the distance between the two places.
- Q.9 Two stones are on the opposite sides of a cliff of 30m high. If the angles of elevation on the top of the cliff observed from the two stones are  $30^\circ$  and  $45^\circ$ , find the distance between two stones.
- Q.10 An angle of elevation of the top of a tower from a point on a plane surface is  $30^\circ$ . On moving 30m near the tower the angle of elevation is  $60^\circ$ . Find the height of the tower.
- Q.11 The angle of depressions from the top of a house on the two points A and B on the same side of the house are  $60^\circ$  and  $45^\circ$  respectively. If distance between A and B is 20m, find the distance of the house from the point A and the height of the house.

- Q.12 If the angle of depressions from the top of a tower on the two places 50m apart are  $45^\circ$  and  $60^\circ$ , find the height of the tower.
- Q.13 The angles of depression from the top of an observation house 200m high on the two boats on the surface of the sea on the same side are  $30^\circ$  and  $45^\circ$ . (i) Find the distance between the two boats and (ii) How far is the farthest boat from the observation house?
- Q.14 The angle of elevations observed from the two places of the earth to the satellite are  $30^\circ$  and  $60^\circ$ . If the distance between two places is 4000km, find the height of the satellite.
- Q.15 The angles of depression on the two places of the ground from the top of 150m high hill are  $30^\circ$  and  $60^\circ$  respectively. Find the distance between them.
- Q.16 From the top of 15m high house, the angle of elevation and the depression of the top and bottom of reflection antenna of the tele-communication house are  $45^\circ$  and  $15^\circ$  respectively. Find the height of the antenna.
- Q.17 From the top of a cliff of 200m high, the angles of elevation and the angle of depression of the top and bottom of a hill are  $45^\circ$  and  $20^\circ$  respectively. Find the height of the hill.
- Q.18 From the top of a cliff of 100m high, the angles of depression of the top and bottom of a tree are  $30^\circ$  and  $52^\circ$  respectively, find the height of the tree.
- Q.19 From the top of a house of 20m high, the angle of elevation and the depression of the top and bottom of a television tower are  $45^\circ$  and  $15^\circ$  respectively. Find the height of the tower.
- Q.20 From the top of a house of 60m high, the angles of depression on the top and bottom of lamppost in front of the house are  $30^\circ$  and  $60^\circ$  respectively. Find the height of the lamppost and the distance between the house and the lamppost.
- Q.21 From the top of a tower, the angle of depression of the bottom of a pole of 20m high is  $60^\circ$  and the angle of depression of the bottom of the tower from the top of the pole is  $22^\circ$ . Find the height of the tower.
- Q.22 A pole is 10m high. The angles of elevation of the top of the clock house from the top and bottom of the pole are  $45^\circ$  and  $22^\circ$  respectively. Find the height of the clock house and the distance between the pole and the clock house.
- Q.23 A pole is 20m high. The angles of elevation of the top of a house from the top and bottom of the pole are  $30^\circ$  and  $45^\circ$  respectively.  
(xxv) What is the height of the house?

(xxvi) What is the distance between the house and the pole?

- Q.24 A 9m high flagstaff stands on the top of a house. If the angle of elevation on the top of flagstaff from any point on the ground is  $60^\circ$  and that from the same point to the top of the house is  $30^\circ$ , find the height of the house.
- Q.25 A tree of 12m high is broken by the wind but not completely separated and its upper part meets the ground at an angle of  $60^\circ$ . Find the height of the broken part and the remaining part of the tree.
- Q.26 A flagstaff is fixed on the top of a tower. If the top of the tower and the flagstaff make angles of depression  $30^\circ$  and  $45^\circ$  respectively on the ground 100, far from the tower, find the height of the flagstaff.
- Q.27 Two lampposts with equal height stand on the either side of a road in 30m wide. The angles of elevation of the top of the posts from a point in the road way between the posts are  $60^\circ$  and  $30^\circ$ . Find the heights of the lampposts and the position of the point.
- Q.28 A tower and a flagstaff fixed on the top of the tower make angles  $45^\circ$  and  $15^\circ$  on a place of 100 ft apart from the tower, find the height of the flagstaff.
- Q.29 A 1.6m high man is standing 4.68m far from the lamppost. If the length of the shadow of the man is 2,88m, find the height of the lamppost.
- Q.30 A 60m high tree is broken by the wind but not completely separated and its upper part meets the ground at an angle of  $30^\circ$ . Find the distance of the point where the top of the tree meets the ground from the foot and also the height at which the tree is broken.
- Q.31 A pole is divided into two parts in the ratio 4: 3. If both the parts of the pole subtend equal angles at a point 50m from the foot of the pole. Find the height of the pole.
- Q.32 A rope dancer was walking on a loose rope tied to the tops of the two posts, each 10m high. When the dancer was 2.5m above the ground, it was found that the stretched pieces of the rope subtends  $30^\circ$  and  $60^\circ$  with the horizontal line parallel to the ground. Find the length of the rope.
- Q.33 The angles of depression and elevation of the top of the a tower 50m high from the top and the bottom of a second tower are  $45^\circ$  and  $30^\circ$  respectively. Find the height of the second tower.
- Q.34 The angle of elevation of an airplane from a point on the ground is  $45^\circ$ . After a flight for 30 seconds, the angle of elevation changes to  $30^\circ$ . If the airplane is flying at a constant height of 6000m, find the speed of the airplane.

- Q.35 A man 2m high standing on the road before a house finds the angles of elevation of the tops of the house and the window to be  $45^\circ$  and  $30^\circ$  respectively. If the window is at a height of 12m, find the height of the house.
- Q.36 The sum of the angles of elevation of the top of a tower from the two points due to the foot of the tower is  $90^\circ$ . If two points are at a distance 40m and 70m from the foot of the tower, find the height of the tower.
- Q.37 The angles of elevation of the top of a tower from the two points at a distance of 'x' and 'y' from the base and in the same straight line with it are complementary. Prove that the height of the tower is  $\sqrt{xy}$ .
- Q.38 From an airplane flying vertically over a straight road, the angles of depression of two consecutive stones on the same side are  $45^\circ$  and  $60^\circ$ . If the height of the airplane is 2.366 km, find the distance between the stones.
- Q.39 The angle of the elevation of the top of an incomplete tower at a point distance from 120, from its base is  $45^\circ$ . How much height must the tower be raised so that its angle of elevation at the same point is  $60^\circ$ ?
- Q.40 The angle of elevation of the top of the an incomplete house as observed from a point on the ground is  $30^\circ$ . If the height of the incomplete house is 30m, how high the house must be raised so that the angle of elevation of the top of the completed house as observed from the same point on the ground is  $45^\circ$ ?
- Q.41 Two towers are 180m apart and the heights of one is three times that of the other. From the middle point of the line joining their feet, the angles of elevation of the tops are found to be complementary. Find the heights of the towers.
- Q.42 There are two posts, one shorter than the other by 5m. The angles of depression of the feet of the shorter and the larger posts from the tops of the larger and the shorter posts are  $20^\circ$  and  $15^\circ$  respectively. Find the heights of the posts and the distance between them.
- Q.43 A ladder 20m long rests against a house at one side of a road at an angle of  $30^\circ$  with the house. When it is turned so that it rests against another house on the other side of a road, it makes an angle of  $45^\circ$  on the house, find the width of the road.
- Q.44 A ladder 6m long reaches a point 6m below the top of a vertical flagstaff. From the foot of the ladder, the angle of elevation of the flagstaff is  $60^\circ$ . Find the height of the flagstaff.
- Q.45 The height of a tower is half the height of the flagstaff on the top. The angle of elevation of the top of the tower as observed from the distance 2m from its foot is  $30^\circ$ . Find the angle of elevation of the top of the flagstaff from the same point.

- Q.46 The angle of elevation of a tower was observed to be  $60^\circ$  from a point. On walking 200m away the point it was found to be  $30^\circ$ . Find the height of the tower.
- Q.47 The angle of elevation of the top of a tower from a point has observed to be  $45^\circ$ . On walking 30m away from that point, it was found to be  $30^\circ$ . Find the height of the tower.
- Q.48 From the top of 21m high cliff, the angles of depression of the top and the bottom of a tower are observed to be  $45^\circ$  and  $60^\circ$  respectively. Find the height of the tower.
- Q.49 A pole is surmounted on its top by a flagstaff. The angles of elevation of the top and the bottom of the flagstaff as observed from a point 30 meters away from the foot of the pole are found to be  $45^\circ$  and  $30^\circ$  respectively. Find the height of the flagstaff.
- Q.50 Two men are on the opposite side of a tower of 30m high. They observed the angles of elevation of the top of the tower and found to be  $30^\circ$  and  $60^\circ$ . Find the distance between them.
- Q.51 The angle of elevation of the top of the tower as observed form the distances of 36m and 16m from the foot of the tower are found to be complementary. Find the height of the tower.
- Q.52 A flagstaff of height 7 meters stands on the top of a tower. The angles subtend by the tower and the flagstaff to the point on the ground are  $45^\circ$  and  $15^\circ$  respectively. Find the height of the tower.
- Q.53 The angle of elevation of the top of a tower as observed form a point on the ground is found to be  $60^\circ$ . On walking 60m away form the point, the angle of elevation was found to be  $45^\circ$ . Find the height of the tower.
- Q.54 A man of 1.68 height observed the angles of elevation of the top of a house and its window form a place and found to be  $45^\circ$  and  $30^\circ$  respectively. If the height of the window form the ground is 11.68m, calculate the height of the house.
- Q.55 From the top of 42 m high cliff, the angle of depression of the top and bottom of a tower are observed to be  $30^\circ$  and  $45^\circ$  respectively. Find the height of the tower.
- Q.56 From a point on the horizontal plane, the angle of elevation of the top of a pillar standing on the same plane was observed and found to be  $60^\circ$  and the angle of elevation of a point 20 below the top of the pillar was found to be  $30^\circ$ . Find the height of the pillar.
- Q.57 A man 1.75 m tall observed the angle of elevation of the top and the angle of depression of the bottom of a house found to be  $45^\circ$  and  $10^\circ$  respectively, find the height of the house.

- Q.58 From the top a cliff 81m high, the angles of depression of the top and the bottom of a tower are found to be  $45^\circ$  and  $60^\circ$  respectively. Find the height of the tower.
- Q.59 From the top of a building 30 m high a man observes two persons sitting on the ground, both due east on the same line at angles of depression of  $45^\circ$  and  $30^\circ$ . How far apart are the two persons?
- Q.60 From the top of a tower the angles of depression of two stones, which are 100m and 300m far from the foot of the tower is complementary. Find the height of the tower if the two stones lie on the same side of the tower and the line joining the stones passes through the base of the tower.
- Q.61 Two poles such that one is double the height of the other and are at a distance of 40m. If the angles of elevation of the top of the poles from a point midway between them are complementary find the height of the poles.
- Q.62 The angle of elevation of the top of a tower from the bottom of a column is  $60^\circ$  and the angle of depression of the top of the column from the top of the tower is  $30^\circ$ . If the height of the column is 36m, find the height of the tower.
- Q.63 The angle of elevation of an airplane from a point A on the ground in  $45^\circ$ . After 20 seconds flight, the angle of elevation changes to  $30^\circ$ . If the airplane is flying at a height of 2 km, find the speed of airplane.
- Q.64 An airplane when 3000m high passes vertically above another airplane at an instant when their angles of elevation at the same observation point are  $60^\circ$  and  $45^\circ$  respectively. How many meters higher is the one than the other.
- Q.65 Two poles are such that one is double in height to the other and are at a distance of 20m. If the angles of elevation of the tops from a point midway between them are complementary, find the height of the poles.
- Q.66 Two poles of equal height stand on either of a roadway which is 10m wide. At a point on the roadway between the poles, the angles of elevation of the tops of the poles are  $60^\circ$  and  $30^\circ$ . Find the heights of the poles and position of the point.
- Q.67 A vertical pole of height 'x' fixed to the ground is surmounted by flagstaff. If both the parts subtend equal angles at a distance 'y' from the base of the pole, show that the height of the flagstaff is  $[x(x^2 + y^2) / (y^2 - x^2)]$ .
- Q.68 A pole is divided at a point in the ratio 1:9. If the two parts of the pole subtend equal angles at a point 20m from the foot of the pole. Find the height of the pole.
- Q.69 The angle of depression of the top of the building of 24m height from the top of a tower is  $30^\circ$  and the angle of elevation of the top of the tower from the foot of the building is  $45^\circ$ . Find the height of the tower.

- Q.70 Two lampposts are of equal height. A man standing mid-way between them observes its elevation of either post to be  $30^\circ$ . After walking 20 m towards one of them, he observes its elevation to be  $45^\circ$ . Find the height of the posts and the distance between the posts.
- Q.71 A ladder 6m long touches a point 6 m below the top of a vertical pole. From the foot of the ladder, the angle of elevation of the pole is  $60^\circ$ . Find the height of the pole.
- Q.72 The angle of depression and elevation of the top of a pole 25 m high from the top and bottom of a tower are  $60^\circ$  and  $30^\circ$  respectively. Find the height of the tower.
- Q.73 A pole  $10\sqrt{3}$  m high subtends an angle  $\theta$  at a point which is 30 m away from the foot of the pole. At another point between the first point and the foot of the pole, the pole subtends an angle  $2\theta$ . Find the distance of the second point from the foot of the pole and also the distance between the two points.
- Q.74 AB is a straight road leading to C, the foot of the pole; B and C are being  $8\sqrt{3}$  m and  $12\sqrt{3}$  respectively far from the point A. If the angle of elevation of the top of pole at B is double of an angle of elevation of A, find the height of the pole.
- Q.75 A man who is 2 meters tall stands at a distance of 4 meters from a lamp post and it is observed that his shadow is 4 meters long. Find the height of the lamp post.
- Q.76 When the top of a building is seen from the top and bottom of a tower, the angles of depression and elevation are  $60^\circ$  and  $30^\circ$  respectively. If the height of the building is 20 m, find the height of the tower.
- Q.77 From the top of a tower 20 m high, the angles of depression of the top and bottom of a building are observed to be  $30^\circ$  and  $60^\circ$  respectively. Find the height of the building.
- Q.78 From the top of a building of 20 m high, the angles of depression of the top and bottom of a pole are observed to be  $45^\circ$  and  $60^\circ$  respectively. Find the height of the pole and the distance between the house and the pole.